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Proton stability in superstring derived models

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ABSTRACT

I discuss the problem of proton decay, from dimension four, five and six operators, in superstring derived standard–like models. I classify the sectors that produce color triplet superfields which may generate proton decay from dimension five and six operators. I show that for two of these sectors there exist a unique superstring doublet–triplet splitting mechanism that projects out the color triplets by means of the GSO projection, while leaving the electroweak doublets in the physical spectrum. I investigate possible proton decay due to additional color triplets in the massless spectrum and due to nonrenormalizable terms. I show that there exist models in which the dimension four, five and six operators, that lead to proton decay, are forbidden by the symmetries of the string models, to all orders of nonrenormalizable terms.

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1. Introduction

LEP precision data provides further support to the validity of the Standard Model at the electroweak scale and possibly to a much higher energy scale. In the last two decades this possibility has been exploited in the development of Grand Unified Theories and Superstring Theories. In general, theories of unification lead to a well known prediction, namely proton decay. Non supersymmetric unified theories are all but ruled out due to proton decay constraints, while their supersymmetric counterparts must accommodate some ad hoc and very stringent symmetries, if they are to survive proton lifetime limits. Supersymmetric models [1], in general, admit proton decay from dimension four, five and six operators [2,3]. Dimension four operators are avoided by imposing R–parity. Dimension six operators, from gauge boson exchange, are suppressed because the unification scale in supersymmetric models is higher than in their nonsupersymmetric counterparts [4]. However, in supersymmetric models dimension five operators from Higgsino exchange are the most problematic. This requires that the Higgsino color multiplets are sufficiently heavy, of the order of 10^{16} GeV. Supersymmetric GUT models must admit some doublet–triplet splitting mechanism, which satisfies these requirements. Although, such a mechanism has been constructed in different supersymmetric GUT models, in general, further assumptions have to be made on the matter content and interactions of the supersymmetric GUT models [5]. An important point of this paper is to show how the doublet–triplet splitting problem may be solved in a class of superstring standard–like models.

Superstring theories [6] lead in their point field theory limit to $N = 1$ space–time supersymmetry and therefore usually suffer from the same problems. In the context of superstring theory the problem is worsen because one cannot impose additional symmetries at will. The desired symmetries must be derived in the massless spectrum of specific string vacua. In many string models the required symmetries are obtained at specific points in their moduli space [7]. However, to produce realistic low energy mass spectrum it is in general necessary to perturb away from the symmetric points in moduli space. In this case the operators that

produce proton decay are generated from nonrenormalizable terms. The nonrenormalizable terms of order N produce effective terms, $\lambda f f b \phi^L \langle\phi\rangle^{N-L}/M^{N-3}$, where $\langle\phi\rangle$ are the VEVs that break the symmetries which forbid the cubic level Baryon violating operators. These VEVs are necessary if we impose a supersymmetric vacuum at the Planck scale [8]. Thus, although the dangerous operators are forbidden at the cubic level of the superpotential they are in general generated from nonrenormalizable terms. To satisfy proton lifetime limits there must exist a mechanism that suppresses the dangerous operators, which is generation independent and which is independent of the particular point in moduli space.

In this paper I study the problem of proton decay in a class of superstring standard–like models [9–12] that are constructed in the free fermionic formulation [16]. I argue that in string models dimension four, baryon and lepton violating, operators are in general generated. The existence of an additional, generation independent, $U(1)$ symmetry which remains unbroken down to some energy scale is sufficient to solve this problem [2,14,15]. I show that in the superstring standard–like models there is a unique doublet–triplet splitting mechanism. The dangerous color triplets are projected out from the physical spectrum by the GSO projection, while the electroweak doublets remain in the physical spectrum. The GSO projections are imposed by requiring modular invariance. The models contain additional color triplets from sectors that are generated by the Wilson line breaking. I show in one specific model that the symmetries of the string model forbid the dangerous dimension five and six operators, from Higgs and Higgsino exchange and to all orders of nonrenormalizable terms.

2. The superstring models

The superstring standard–like models are derived in the free fermionic formulation [16]. In this formulation all the degrees of freedom needed to cancel the conformal anomaly are represented in terms of internal free fermions propagating on the string world–sheet. Under parallel transport around a non-contractible loop, the fermionic states pick up a phase. Specification of the phases for all world–

sheet fermions around all noncontractible loops contributes to the spin structure of the model. The possible spin structures are constrained by string consistency requirements (e.g. modular invariance). A model is constructed by choosing a set of boundary condition vectors, which satisfies the modular invariance constraints. The basis vectors, b_k, span a finite additive group $\Xi = \sum_k n_k b_k$ where $n_k = 0, \cdots, N_{z_k} - 1$. The physical massless states in the Hilbert space of a given sector $\alpha \in \Xi$, are obtained by acting on the vacuum with bosonic and fermionic operators and by applying the generalized GSO projections.

The superstring standard–like models are generated by a basis of eight vectors of boundary conditions for all the world–sheet fermions. The first five vectors in the basis consist of the NAHE set, $\{\mathbf{1}, S, b_1, b_2, b_3\}$ [17,10]. The gauge group after the NAHE set is $SO(10) \times SO(6)^3 \times E_8$ with $N = 1$ space–time supersymmetry. The vector S is the supersymmetry generator and the superpartners of the states from a given sector α are obtained from the sector $S + \alpha$. The vectors b_1, b_2 and b_3 correspond to the three twisted sectors in the corresponding orbifold formulation. In addition to the first five vectors the basis contains three additional vectors that correspond to Wilson line in the orbifold language. The additional vectors distinguish between different models and determine their low energy properties.

The NAHE set divides the 44 right–moving and 20 left–moving real internal fermions in the following way: $\bar{\psi}^{1,\cdots,5}$ are complex and produce the observable $SO(10)$ symmetry; $\bar{\phi}^{1,\cdots,8}$ are complex and produce the hidden E_8 gauge group; $\{\bar{\eta}^1, \bar{y}^{3,\cdots,6}\}$, $\{\bar{\eta}^2, \bar{y}^{1,2}, \bar{\omega}^{5,6}\}$, $\{\bar{\eta}^3, \bar{\omega}^{1,\cdots,4}\}$ give rise to the three horizontal $SO(6)$ symmetries. The left–moving $\{y, \omega\}$ states are divided to, $\{y^{3,\cdots,6}\}$, $\{y^{1,2}, \omega^{5,6}\}$, $\{\omega^{1,\cdots,4}\}$. The left–moving $\chi^{12}, \chi^{34}, \chi^{56}$ states carry the supersymmetry charges. Each sector b_1, b_2 and b_3 carries periodic boundary conditions under $(\psi^\mu | \bar{\psi}^{1,\cdots,5})$ and one of the three groups: $(\chi_{12}, \{y^{3,\cdots,6} | \bar{y}^{3,\cdots 6}\}, \bar{\eta}^1)$, $(\chi_{34}, \{y^{1,2}, \omega^{5,6} | \bar{y}^{1,2} \bar{\omega}^{5,6}\}, \bar{\eta}^2)$ and $(\chi_{56}, \{\omega^{1,\cdots,4} | \bar{\omega}^{1,\cdots 4}\}, \bar{\eta}^3)$. The division of the internal fermions is a reflection of the underlying $Z_2 \times Z_2$ orbifold compactification. The set of internal fermions $\{y, \omega | \bar{y}, \bar{\omega}\}^{1,\cdots,6}$ corresponds to the left–right symmetric conformal field theory of the heterotic string, or to the six dimensional compactified manifold in a bosonic

formulation. This set of left–right symmetric internal fermions plays a fundamental role in the determination of the low energy properties of the superstring standard–like models. In particular, it plays an important role in the superstring doublet–triplet splitting mechanism.

The observable gauge group after application of the generalized GSO projections is $SU(3)_C \times U(1)_C \times SU(2)_L \times U(1)_L \times U(1)^3 \times U(1)^n$ *. The weak hypercharge is given by $U(1)_Y = \frac{1}{3}U(1)_C + \frac{1}{2}U(1)_L$ and the orthogonal combination is given by $U(1)_{Z'} = U(1)_C - U(1)_L$. The first three additional $U(1)$ symmetries arise from the world–sheet complex fermions, $\bar{\eta}^j$, $(j = 1, 2, 3)$. The additional $U(1)^n$ symmetries arise from complexifying two right–moving real fermions from the set $\{\bar{y}, \bar{\omega}\}^{1,\cdots,6}$. For each right–moving gauged $U(1)$ symmetry there is a corresponding left–moving global $U(1)$ symmetry. Alternatively, a left–moving real fermion can be paired with a right–moving real fermion to form an Ising model operator [21]. The hidden gauge group after application of the generalized GSO projections is $SU(5)_H \times SU(3)_H \times U(1)^2$. The $U(1)$ symmetries in the hidden sector, $U(1)_7$ and $U(1)_8$, correspond to the world–sheet currents $\bar{\phi}^1\bar{\phi}^{1^*} - \bar{\phi}^8\bar{\phi}^{8^*}$ and $-2\bar{\phi}^j\bar{\phi}^{j^*} + \bar{\phi}^1\bar{\phi}^{1^*} + 4\bar{\phi}^2\bar{\phi}^{2^*} + \bar{\phi}^8\bar{\phi}^{8^*}$ respectively, where summation on $j = 5, \cdots, 7$ is implied.

The massless spectrum of the standard–like models contain three chiral generations from the sectors b_1, b_2 and b_3 with charges under the horizontal symmetries. Three generations from the sectors b_1, b_2 and b_3 are common to all the free fermionic standard–like models. For example in the model of Ref. [13] we have,

$$(e_L^c + u_L^c)_{\frac{1}{2},0,0,\frac{1}{2},0,0} + (d_L^c + N_L^c)_{\frac{1}{2},0,0,\frac{1}{2},0,0} + (L)_{\frac{1}{2},0,0,-\frac{1}{2},0,0} + (Q)_{\frac{1}{2},0,0,-\frac{1}{2},0,0}, \quad (1a)$$
$$(e_L^c + u_L^c)_{0,\frac{1}{2},0,0,\frac{1}{2},0} + (N_L^c + d_L^c)_{0,\frac{1}{2},0,0,\frac{1}{2},0} + (L)_{0,\frac{1}{2},0,0,-\frac{1}{2},0} + (Q)_{0,\frac{1}{2},0,0,-\frac{1}{2},0}, \quad (1b)$$
$$(e_L^c + u_L^c)_{0,0,\frac{1}{2},0,0,\frac{1}{2}} + (N_L^c + d_L^c)_{0,0,\frac{1}{2},0,0,\frac{1}{2}} + (L)_{0,0,\frac{1}{2},0,0,-\frac{1}{2}} + (Q)_{0,0,\frac{1}{2},0,0,-\frac{1}{2}}. \quad (1c)$$

* $U(1)_C = \frac{3}{2}U(1)_{B-L}$ and $U(1)_L = 2U(1)_{T_{3_R}}$.

where

$$e_L^c \equiv [(1,\frac{3}{2});(1,1)]; \quad u_L^c \equiv [(\bar{3},-\frac{1}{2});(1,-1)]; \quad Q \equiv [(3,\frac{1}{2});(2,0)] \qquad (2a,b,c)$$

$$N_L^c \equiv [(1,\frac{3}{2});(1,-1)]; \quad d_L^c \equiv [(\bar{3},-\frac{1}{2});(1,1)]; \quad L \equiv [(1,-\frac{3}{2});(2,0)] \qquad (2d,e,f)$$

of $SU(3)_C \times U(1)_C \times SU(2)_L \times U(1)_L$ The vectors b_1, b_2, b_3 are the only vectors in the additive group Ξ which give rise to spinorial 16 of $SO(10)$.

The vector 2γ breaks the E_8 symmetry at the level of the NAHE set to $SO(16)$. The sectors $b_j + 2\gamma$ produce 16 representation of $SO(16)$, which, decompose under the final hidden gauge group after application of the α, β and γ projections. Like the sectors b_j, the sectors $b_j+2\gamma$ are common to all the free fermionic standard–like models. For example in the model of Ref. [13] we have,

$$(V_1+T_1)_{0,\frac{1}{2},\frac{1}{2},\frac{1}{2},0,0} + (\bar{V}_1+\bar{T}_1)_{0,\frac{1}{2},\frac{1}{2},-\frac{1}{2},0,0}, \qquad (3a)$$

$$(V_2+T_2)_{\frac{1}{2},0,\frac{1}{2},0,\frac{1}{2},0} + (\bar{V}_2+\bar{T}_2)_{\frac{1}{2},0,\frac{1}{2},0,-\frac{1}{2},0}, \qquad (3b)$$

$$(V_3+T_3)_{\frac{1}{2},\frac{1}{2},0,0,0,\frac{1}{2}} + (\bar{V}_3+\bar{T}_3)_{\frac{1}{2},\frac{1}{2},0,0,0,-\frac{1}{2}} \qquad (3c)$$

There are several sectors that can, a priori, produce electroweak Higgs doublets and color triplets. The first is the Neveu–Schwarz sector. The Neveu–Schwarz sector correspond to the untwisted sector in the orbifold formulation. At the level of the NAHE set the Neveu–Schwarz produces three vector 10 representation of $SO(10)$, which decompose as $5+\bar{5}$ under $SU(5)$. These are obtained by acting on the vacuum with $\chi^j_{\frac{1}{2}}\bar{\psi}^{1\cdots5}_{\frac{1}{2}}\bar{\eta}^j_{\frac{1}{2}}$, $(j=1,2,3)$, where $\chi^j_{\frac{1}{2}}$, $(j=1,2,3)$, denotes one of the pairs $\chi^{12}_{\frac{1}{2}}, \chi^{34}_{\frac{1}{2}}, \chi^{56}_{\frac{1}{2}}$ respectively.

The second sector that may produce color triplet supermultiplets is the sector $\zeta = b_1 + b_2 + \alpha + \beta$. The unique property of this vector is that it does not have periodic world–sheet fermions under $SO(10)\times E_8$ and it has $\zeta_R\cdot\zeta_R = \zeta_L\cdot\zeta_L = 4$. The massless states are obtained by acting on the vacuum with one right–moving fermionic oscillator. The states in this sector transform only under the observable

gauge group. The vector combination that produces these additional color triplets is not generic to all the free fermionic models. For example, in the model of Ref. [9] such a vector combination does not exist in the additive group. However, requiring realistic low energy mass spectrum necessitates the presence of such a vector combination in the additive group [10,15].

The Neveu–Schwarz sector and the sector $\zeta = b_1 + b_2 + \alpha + \beta$ may produce the most dangerous Higgs color triplet representations. The reason is that the states from these two sectors transform solely under the observable gauge group. In addition to these two sectors, two additional type of sectors may produce additional color triplets. These sectors are obtained from combination of the vectors $\{b_1, b_2, b_3, \alpha, \beta\} \pm \gamma$ or $+2\gamma$. The two type of sectors are distinguished by the product $\alpha_R \cdot \alpha_R = 6, 8$. For the first type, $\alpha_R \cdot \alpha_R = 6$ and the color triplets are obtained by acting on the vacuum with a right–moving fermionic oscillator, while in the second type, the vacuum is composed entirely of Ramond vacua. Table 1 summarizes the additional color triplets in the model of Ref. [13].

3. The superstring doublet–triplet splitting mechanism

The additional vectors $\{\alpha, \beta, \gamma\}$ break the gauge symmetry and reduce the number of generations to three. The final observable and hidden gauge groups depend on the assignment of boundary conditions, in these vectors, to the sets $\{\bar\psi^{1,\cdots,5}, \bar\eta^{1,2,3}\}$ and $\{\bar\phi^{1,\cdots 8}\}$, respectively. The final number of generations depends on the assignment of boundary conditions for the set of internal fermions $\{y, \omega|\bar y, \bar\omega\}^{1,\cdots,6}$. This set of internal fermions corresponds to the six compactified dimension in a bosonic formulation or to the left–right symmetric internal conformal field theory. Whether or not the massless spectrum contains color Higgs triplets depends as well on the assignment of boundary conditions to the set of left–right symmetric internal fermions, $\{y, \omega|\bar y, \bar\omega\}^{1,\cdots,6}$.

First, I examine the $5 + \bar 5$ representations from the Neveu–Schwarz sector. In this case we observe a doublet–triplet splitting mechanism that is correlated with

the presence of additional horizontal $U(1)$ symmetries that arise from the set of left–right symmetric internal fermions. The additional $U(1)$ symmetries arise from pairing real right–moving fermions to form a complex fermion. Alternatively a right–moving real fermion can be paired with a left–moving real fermion to form an Ising model operator [21]. The assignment of boundary conditions in all vectors is identical for these pairs, and there exists at least one vector combination that separates them from all the other internal fermions. For every right–moving pair that produces a gauged $U(1)$ current there is a symmetric left–moving pair that produces a global $U(1)$ symmetry. These pairs of right and left–moving internal fermions guarantee that the color triplets, D_j and $\bar{D}_j$ from the Neveu–Schwarz sector are projected out and that the Higgs doublets, h_j and $\bar{h}_j$, remain in the massless spectrum. A selection rule is observed in the application of the GSO projection, α, which breaks the $SO(10)$ symmetry to $SO(6)\times SO(4)$. I denote by $\alpha_L(b_j)$, and $\alpha_R(b_j)$, the intersection of the periodic boundary conditions , in the vectors α and b_j, of the sets of internal fermions $\{y,\omega\}_L$, and $\{\bar{y},\bar{\omega}\}_R$, respectively. The superstring doublet–triplet selection rule then says:

If $|\alpha_L(b_j)-\alpha_R(b_j)|=0$ then the electroweak doublets, h_j, are projected out and the color triplets, D_j, remain in the physical spectrum. if $|\alpha_L(b_j)-\alpha_R(b_j)|=1$ then the color triplets are projected out and the electroweak doublets remain in the physical spectrum.

I now prove this doublet–triplet selection rule. The only states that contribute to the physical spectrum are those that satisfy the generalized GSO projections,

$$\left\{e^{i\pi(b_iF_\alpha)}-\delta_\alpha c^*\begin{pmatrix}\alpha\\ b_i\end{pmatrix}\right\}|s\rangle=0 \qquad (4a)$$

with

$$(b_iF_\alpha)\equiv\{\sum_{\substack{real+complex\\ left}}-\sum_{\substack{real+complex\\ right}}\}(b_i(f)F_\alpha(f)), \qquad (4b)$$

where $F_\alpha(f)$ is a fermion number operator counting each mode of f once (and if f is complex, f^* minus once). For periodic fermions the vacuum is a spinor in

order to represent the Clifford algebra of the corresponding zero modes. For each periodic complex fermion f, there are two degenerate vacua $|+\rangle$, $|-\rangle$, annihilated by the zero modes f_0 and f_0^* and with fermion number $F(f) = 0, -1$ respectively. In Eq. (7a), $\delta_\alpha = -1$ if ψ^μ is periodic in the sector α, and $\delta_\alpha = +1$ if ψ^μ is antiperiodic in the sector α.

The $5 + \bar{5}$ representations from the Neveu–Schwarz sector are of the form,

$$h_j \equiv \chi^j_{\frac{1}{2}} \bar{\psi}^{1,\cdots,5^*}_{\frac{1}{2}} \bar{\eta}^j_{\frac{1}{2}}, \tag{5}$$

The GSO projection coefficients for the Neveu–Schwarz sector are

$$c^* \begin{pmatrix} NS \\ \alpha \end{pmatrix} = \delta_\alpha = exp(i\pi\alpha(\psi^\mu)) \tag{6}$$

and $\delta_{NS} = +1$. By operating the GSO projection, Eq. (4), of the vector α on the representations, Eq. (5), the following equation is obtained,

$$\alpha(\chi_j) + \alpha(\bar{\psi}^{1,\cdots,3}) + \alpha(\bar{\psi}^{4,5}) + \alpha(\bar{\eta}_j) = \alpha(\psi^\mu) \; mod \; 2. \tag{7}$$

The modular invariance constraint on the product of two basis vectors gives

$$\alpha \cdot b_j = \alpha(\psi^\mu) + \alpha(\chi^j) + (\alpha_L(b_j) - \alpha_R(b_j)) - \sum_{i=1}^{3} \alpha(\bar{\psi}^{1,\cdots,3}) - \alpha(\bar{\eta}_j) = 0 \; mod \; 2, \tag{8}$$

where $(\alpha_L(b_j) - \alpha_R(b_j))$ is the difference of the left–right symmetric part of $\alpha \cdot b_j$. In the vector α, $\alpha(\bar{\psi}^{1,\cdots,3}) = 1$ and $\alpha(\bar{\psi}^{4,5}) = 0$. There are four possible boundary conditions for the pair (ψ^μ, χ^j) in the vector α, $\alpha(\psi^\mu, \chi^j) = \{(1,1);(1,0);(0,1);(0,0)\}$. I take, for example, the first case $\alpha(\psi^\mu, \chi^j) = (1,1)$. In

this case, for $|(\alpha_L(b_j) - \alpha_R(b_j))| = 0$, Eqs. (7,8) reduce to

$$\alpha(\bar{\psi}^{1,\cdots,3}) + \alpha(\bar{\psi}^{4,5}) + \alpha(\bar{\eta}_j) = 0 \; mod \; 2. \tag{9}$$

and

$$-\sum_{i=1}^{3} \alpha(\bar{\psi}^{1,\cdots,3}) - \alpha(\bar{\eta}_j) = 0 \; mod \; 2. \tag{10}$$

Because $\sum_{i=1}^{3} \alpha(\bar{\psi}^{1,\cdots,3}) = 3$, from Eq. (10) follows $\alpha(\bar{\eta}_j) = 1$. Then Eq. (9) can be satisfied for the triplets but cannot be satisfied for the doublets. Therefore the electroweak doublets are projected out while the color triplets remain in the massless spectrum.

On the other hand, for $|(\alpha_L(b_j) - \alpha_R(b_j))| = 1$, Eq. (8) reduces to

$$-\sum_{i=1}^{3} \alpha(\bar{\psi}^{1,\cdots,3}) - \alpha(\bar{\eta}_j) = 1 \; mod \; 2, \tag{11}$$

and with $\alpha(\psi^\mu, \chi^j) = (1,1)$ Eq. (7) remains as in Eq. (9). Therefore in this case, from Eq. (11) follows $\alpha(\bar{\eta}_j) = 0$. In this case the triplets cannot satisfy Eq. (9) and are therefore projected out, while the doublets remain in the massless spectrum. In a similar way this selection rule can be shown to hold for the other choices of boundary conditions, in the vector α, for the pair (ψ^μ, χ^j). To summarize, the constraint

$$|\alpha_L(b_j) - \alpha_R(b_j)| = 1 \quad , \quad (j = 1, 2, 3), \tag{12}$$

guarantees that the Neveu–Schwarz color triplets, D_j, are projected out and that the electroweak doublets remain in the massless spectrum.

To illustrate this dependence I consider the models in tables 2, 3, 4 and 5. In the models of tables 2 and 3, the three horizontal $(U(1)_\ell; U(1)_r)$ symmetries, which correspond to the world-sheet currents $(y^3y^6; \bar{y}^3\bar{y}^6)$, $(y^1\omega^5; \bar{y}^1\bar{\omega}^5)$ and $(\omega^2\omega^4; \bar{\omega}^2\bar{\omega}^4)$, guarantee that the Higgs doublets h_1, $\bar{h}_1$, h_2, $\bar{h}_2$ and h_3, $\bar{h}_3$ remain in the massless

spectrum, and that the color triplets are projected out. In the model of table 4 all the real fermions are paired to form Ising model operators and there are no additional $U(1)$ symmetries beyond $U(1)_{r_j}$ ($j = 1, 2, 3$). All the Higgs doublets from the Neveu–Schwarz sector are projected out. In this case the Higgs triplets D_1, $\bar{D}_1$, D_2, $\bar{D}_2$ and D_3, $\bar{D}_3$ remain in the massless spectrum. In model 5 we have only one additional horizontal $(U(1)_\ell; U(1)_r)$ symmetry which corresponds to the world–sheet currents $(\omega^2\omega^3; \bar{\omega}^2\bar{\omega}^3)$. Therefore in this model only one pair of Higgs doublets from the Neveu–Schwarz sector, h_3, $\bar{h}_3$, remains in the massless spectrum after the GSO projections. In this case we obtain the color triplets D_1, $\bar{D}_1$ and D_2, $\bar{D}_2$.

Next I turn to the sector $S + b_1 + b_2 + \alpha + \beta$. This sector produces additional states that transform solely under the observable sector. In particular it can give rise to additional electroweak doublets and color triplets. To obtain realistic low energy phenomenology, it is essential that such a vector combination exists in the additive group. The color triplets from this sector may cause problems with proton lifetime constraints. However, a similar doublet–triplet splitting mechanism works for this sector as well. There exist choices of boundary conditions for the set of left–right symmetric internal fermions, $\{y, \omega | \bar{y}, \bar{\omega}\}^{1,\cdots,6}$, for which the triplets are projected out and the doublets remain in the massless spectrum. For example, in the model of Ref. [11] (table 2) this sector produces one pair of electroweak doublets and one pair of color triplets.

$$h_{45} \equiv [(1,0);(2,-1)]_{-\frac{1}{2},-\frac{1}{2},0,0,0,0} \quad D_{45} \equiv [(3,-1);(1,0)]_{-\frac{1}{2},-\frac{1}{2},0,0,0,0} \qquad (13a,b)$$

while in the model of Ref. [12,13] (table 3) this sector produces two pairs of electroweak doublets,

$$h_{45} \equiv [(1,0);(2,-1)]_{\frac{1}{2},\frac{1}{2},0,0,0,0} \quad h'_{45} \equiv [(1,0);(2,-1)]_{-\frac{1}{2},-\frac{1}{2},0,0,0,0} \qquad (14a,b)$$

and all the color triplets, from the Neveu–Schwarz sector and the sector $b_1 + b_2 + \alpha + \gamma$, are projected from the physical spectrum by the GSO projections. As

is evident from tables 2 and 3, the two models differ only by the assignment of boundary conditions to the set of internal fermions, $\{y,\omega|\bar{y},\bar{\omega}\}^{1,\cdots,6}$. The simplicity and elegance of the superstring doublet–triplet splitting mechanism is striking. There is no need for exotic representations of high dimensionality as in minimal $SU(5)$ extension of the Standard Model [5]. Moreover, the superstring doublet–triplet splitting mechanism does not depend on additional assumptions on Yukawa couplings as is required in all GUT doublet–triplet splitting mechanism. In the superstring doublet–triplet splitting mechanism the dangerous color triplets simply do not exist in the massless spectrum. In the next section I investigate the implications on proton decay.

4. Proton decay

In the most general supersymmetric standard model the dimension four operators, $\eta_1 u_L^C d_L^C d_L^C + \eta_2 d_L^C Q L$, mediate instantaneous proton decay if η_1 and η_2 are both large. Traditionally in supersymmetric models, one imposes R symmetries on the spectrum to avoid this problem. In the context of superstring theories these discrete symmetries are usually not found [14]. These dimension four operators are forbidden if the gauge symmetry of the Standard Model is extended by a $U(1)$ symmetry, which is a combination of, $B-L$, baryon minus lepton number, and T_{3_R}, and is exactly the additional, generation independent, $U(1)$ symmetry that is derived in the superstring standard–like models. However, the dimension four operators may still appear from nonrenormalizable terms. In terms of $SO(10)$ representations the effective dimension four operators are obtained from a 16^4 operator, if one of the 16 obtains a VEV. In terms of standard model multiplets the dangerous operators are

$$\eta_1 (u_L^C d_L^C d_L^C N_L^c)\Phi + \eta_2 (d_L^C Q L N_L^c)\Phi, \tag{15}$$

where N_L^c is the Standard Model singlet in the 16 of $SO(10)$. Φ is a combination of fields which fixes the string selection rules and gets a VEV of $O(M/10)$, where

$M = M_{Pl}/2\sqrt{8\pi}$. From Eq. (15) it is seen that the ratio $\langle N_L^c\rangle/M$ controls the rate of proton decay. While in the standard–like models we may impose $\langle N_L^c\rangle \equiv 0$, or $\langle N_L^c\rangle$ smaller than some value, in superstring models that are based on an intermediate GUT symmetry the problem is more acute as $\langle N_L^c\rangle$ is necessarily used to break the GUT symmetry. A search through nonrenormalizable terms shows that terms of the form of Eq. (15) are in general generated in string models [20,15].

To guarantee that the dimension four operators are sufficiently suppressed we must ascertain that other $U(1)_{Z'}$ breaking VEVs cannot generate them. In addition to N_L^c the massless spectrum of the superstring standard–like models contain neutral states with fractional $U(1)_{Z'}$. To produce the $U(1)_{Z'}$ charge of N_L^c two such states have to be combined. The possible terms must have the form

$$u_L^c d_L^c d_L^c H H \phi^n \quad \text{and} \quad Q L d_L^c H H \phi^n \tag{16}$$

where $\langle H\rangle$ breaks $U(1)_{Z'}$ and ϕ^n is a string of Standard Model singlets that fixes the remaining string selection rules. For example, in the model of Ref. [13] all the states with fractional $U(1)_{Z'}$ charge transform as 3 and $\bar{3}$ of the hidden $SU(3)$ group (see table 1). Therefore, in this model invariance under the hidden Abelian and non–Abelian gauge groups, and under $U(1)_{Z'}$, forbid the formation of terms of the form of Eq. (16) to all orders of nonrenormalizable terms. However, in general, such terms can be formed. For example in the model of Ref. [11] such terms appear at order $N = 8$ and will not be enumerated here. They contain a suppression factor of $(\Lambda_{Z'}/M)^2$ and are therefore sufficiently suppressed if, for example, $\Lambda_{Z'} \leq 10^{12}\ GeV$.

Next I turn to proton decay from dimension five operators. The dangerous dimension five operators are:

$$QQQh, \qquad QQQL, \qquad d_L^c u_L^c u_L^c e_L^c \tag{17}$$

The first operator is forbidden by $U(1)_{Z'}$ charge conservation. Tagging to it $\bar{N}_L^c$, from the $\bar{16}$ of $SO(10)$ renders it invariant under $U(1)_{Z'}$. In the superstring

standard–like models the $\bar{16}$ of $SO(10)$ is not present in the massless spectrum. One may still form combination states of states with fractional $U(1)_{Z'}$ charge that effectively produce the same charge as $\bar{N}_L^c$. However, this introduces an additional suppression by $\Lambda_{Z'}/M$. Therefore, this operators is suppressed by at least $\Lambda_{Z'}^2/M^3$. Thus, even for rather large $\Lambda_{Z'} \sim 10^{15}$ GeV, this operator is harmless. Therefore, in the superstring standard–like models there are two, potentially dangerous, Baryon violating dimension five operators, $QQQL$ and $d_L^c u_L^c u_L^c e_L^c$. The second operator does not contribute to proton decay [19]. Therefore, we are left with a single operator, $QQQL$.

There are two possible ways in which these operators could be generated. One is through triplet Higgsino exchange. In this case the dimension five operators are obtained from the tree level operators

$$LQ\bar{D},\ u_L^c e_L^c D,\ QQD,\ u_L^c d_L^c \bar{D},\ d_L^c N_L^c D,\ D\bar{D}\phi \tag{18}$$

where D are the color triplets in the 5 of $SU(5)$. Alternatively the dimension five operators may be induced by exchange of heavy string modes. To insure that dangerous dimension five operators are not induced we must check that both the terms in Eq. (18) are suppressed and that the dimension five operators are not induced from nonrenormalizable terms at the string level. In general, if massless triplets from the Neveu–Schwarz sector or the sector $b_1 + b_2 + \alpha + \beta$ exist in the massless spectrum then then the terms in Eq. (18) are obtained either at the cubic level of the superpotential or from higher order nonrenormalizable terms. For example in the model of table 4 (the massless spectrum and quantum numbers are given in Ref. [10]), we obtain at the cubic level,

$$\begin{aligned} &u_{L_1}^c e_{L_1}^c D_1,\ d_{L_1}^c N_{L_1}^c D_1,\ u_{L_2}^c e_{L_2}^c D_2,\ d_{L_1}^c N_{L_2}^c D_1, \\ &\quad D_1 \bar{D}_2 \bar{\Phi}_{12},\ \bar{D}_1 D_2 \Phi_{12} \end{aligned}$$

while in the model of Ref. [11] we obtain at the quartic order,

$$Q_1 Q_1 D_{45} \Phi_1^+,\ Q_2 Q_2 D_{45} \bar{\Phi}_2^-,\ u_1^c e_1^c \bar{D}_{45} \bar{\Phi}_1^-,\ u_2^c e_2^c D_{45} \bar{\Phi}_2^+,\ d_1^c N_1^c D_{45} \Phi_1^+,\ d_2^c N_2^c D_{45} \bar{\Phi}_2^-,$$

$u_2^c e_2^c H_{21} H_{26},\ Q_2 L_2 H_{21} H_{26},$

In general, it is expected that any term that is not forbidden by some symmetry will be generated at some order and therefore may cause problems with proton decay. The validity of the models that contain the dangerous Higgs triplets then rests upon the ability to find flat directions that suppress the dangerous operators. This approach is not very appealing as the choice of flat directions depends also on other phenomenological requirements. Furthermore, if the color triplets are not sufficiently heavy the the mixing between the two Higgs triplets must be suppressed. Otherwise, effective dimension five operators may result from D and $\bar{D}$ exchange.

In the superstring standard–like models there exist a more elegant solution to the problem with dimension five operators from Higgsino exchange. The color triplets that may couple to the Standard Model multiplets are projected out from the massless spectrum by means of the GSO projections. There are no dimension five operators from Higgsino exchange, simply because there are no color Higgs triplets that can produce them. The decisiveness and elegance of the superstring doublet–triplet splitting mechanism can only be highlighted when compared to the proposed solutions in GUT models. In the minimal $SU(5)$ the doublet–triplet splitting mechanism rests upon the existence of very large representations, and then assumes a potential that insures that all the extra matter becomes supermassive. In the flipped $SU(5)$ model there is an elegant doublet–triplet splitting in which the Higgs triplets receive GUT scale mass by coupling them to the triplets in the representations that are used to break the GUT symmetry. These representations are 10 and $\bar{1}0$ of $SU(5)$ that are embedded in the 16 and $\bar{1}6$ of $SO(10)$. The couplings $\lambda_4 HHh + \lambda_5 \bar{H}\bar{H}\bar{h}$ then give large Dirac masses to D and $\bar{D}$. However, also there the mechanism depends on additional assumptions on the coupling λ_4 and λ_5. Therefore, more assumptions are made. In contrast, in the superstring standard–like models no such assumptions are needed. The dangerous Higgs color triplets are simply not there.

I now examine the dimension five operators in the models of Ref. [12,13]. In these models the triplets from the Neveu–Schwarz as well as the triplets from the sector $b_1 + b_2 + \alpha + \beta$ are projected out from the physical spectrum due to the assignment of boundary conditions in the vectors α, β and γ. The sector $b_1 + b_2 + \alpha + \beta$ produces two pairs of electroweak doublets rather than a pair of triplets and a pair of doublets. Thus, in this subclass of superstring standard–like models, there are no color triplets from the Neveu–Schwarz sector, nor from the sector $b_1 + b_2 + \alpha + \beta$. We have also to examine the interactions of the Standard Model states with additional triplets in the massless spectrum. The models of Ref. [12,13] contain additional triplets from sectors that arise due to Wilson line breaking. The number of such triplets is maximized in the model of Ref. [13]. Therefore, in what follows, I focus on this model.

The additional triplets and their quantum numbers under the right–moving $U(1)$ symmetries are given in table 1. The type of correlators that have to be checked are of the form $b_i b_j D\phi^n$, where b_i and b_j represent states from the sectors b_i and b_j, D are the additional color triplets, and ϕ^n is a string of Standard Model singlets. For the first two pairs of color triplets from the sectors $b_{1,2}+b_3+\alpha+\beta$, the operators $b_i b_j D$ are invariant under the weak hypercharge. However, they break $U(1)_{Z'}$ because $Q_{Z'}(D) = (1/2)Q_{Z'}(D_{45})$. Thus, D has one half the $U(1)_{Z'}$ charge of the triplets from the Neveu–Schwarz and $b_1+b_2+\alpha+\beta$ sectors. Therefore, all the operators in Eq. (9), with D being a triplet from one of the sectors $b_{1,2}+b_3+\alpha+\beta$, break $U(1)_{Z'}$. Thus, the string $\langle\phi\rangle^n$ contains a $U(1)_{Z'}$ breaking VEV. However, in these model all the available Standard Model singlets with nontrivial $U(1)_{Z'}$ charge transform as 3 and $\bar{3}$ of the Hidden $SU(3)$ gauge group (see table 1). The $U(1)_{Z'}$ charges of the hidden $SU(3)$ triplets are $\pm 5/4$. The $U(1)_{Z'}$ charges of the color triplets from the exotic "Wilson line" sectors are $\pm 1/4$ (see table 1). The last pair of color triplets has "fractional" weak hypercharge $Q_Y = \pm 1/12$. Therefore, terms of the form of Eq. (18), with D being a triplet from one of the exotic "Wilson line" sectors, cannot be formed in this model. Therefore, in this model proton decay from dimension five or six operators due to Higgsino or Higgs exchange is

suppressed.

Next I show that the dimension five operators cannot be generated by exchange of heavy string modes. I show that the dangerous dimension five operators are forbidden to all orders of nonrenormalizable terms. This follows from the charges of the states from the sectors b_j under the horizontal $U(1)_{r_{1,\cdots,6}}$ (see Eq. (1)). All the states from the sectors b_j have charge 1/2 under $U(1)_{r_j}$ while under $U(1)_{r_{j+3}}$ we have

$$U(1)_{r_{j+3}}(Q,L)=-\frac{1}{2}\ \ \text{and}\ \ U(1)_{r_{j+3}}(d_L^c,u_L^c,e_L^c)=\frac{1}{2}$$

Consider the symmetry $U(1)'=U(1)_4+U(1)_5+U(1)_6$. Under this symmetry, $U(1)'(Q_j)=-1/2$ and $U(1)'(L_j)=-1/2$. The charge of the correlator $QQQL$ under $U(1)'$ is -2. The only additional states in the massless spectrum with, $U(1)'\neq 0$ are the states from the sectors $b_j+2\gamma$. However,

$$U(1)'(3_j)=-U(1)'(\bar{3}_j)\ \ \text{and}\ \ U(1)'(5_j)=-U(1)'(\bar{5})_j\ .$$

In any correlator we can only have $\langle 3\bar{3}\rangle$ or $\langle 5\bar{5}\rangle$, to respect invariance under the Abelian and non–Abelian hidden gauge groups. Therefore the correlator $QQQL$ cannot be invariant under $U(1)'$. Similarly, the total charge of $d_L^cu_L^cu_L^ce_L^c$ under $U(1)'$ is $+2$. Therefore the correlator $d_L^cu_L^cu_L^ce_L^c$ also cannot be invariant under $U(1)'$. This completes the proof that there no dimension five operators in this model. From similar considerations it is also seen that, while the operator $QLd_L^cN_L^c$ is allowed, the operator $u_L^cd_L^cd_L^cN_L^c$ is forbidden by the $U(1)'$ symmetry. Therefore, in this model there is no proton decay from dimension four, five and six operators.

To summarize, in the case of the standard–like models that contain color triplets from the Neveu–Schwarz or $b_1+b_2+\alpha+\beta$ sectors, the problem with proton decay is similar to the problem encountered in traditional SUSY GUT models. One must find specific choices of flat directions for which the dangerous operators are suppressed. However, there is a class of superstring standard–like models in which the color triplets from the Neveu–Schwarz sector and from the sector $b_1+b_2+\alpha+\beta$

are projected out by the GSO projections. In this class of models, proton decay from triplet Higgsino or triplet Higgs exchange is forbidden. The right–moving gauged $U(1)$ symmetries forbid the formation of dimension five operators from nonrenormalizable terms, to all orders. Moreover, the same symmetries also forbid proton decay from effective dimension four operators to all orders of nonrenormalizable terms. Thus, in this class of models there cannot be proton decay from dimension four, five and six operators, to all orders of nonrenormalizable terms.

REFERENCES

1. H. P. Nilles, For reviews see, H.P. Nilles, Phys. Rep. **110** (1984) 1; D.V. Nanopoulos and Lahanas, Phys. Rep. **145** (1987) 1.
2. S. Weinberg, Phys. Rev. D **26** (1982) 475.
3. S. Sakai and T. Yanagida, Nucl. Phys. B **197** (1982) 533; R. Arnowitt, A.H. Chamsdine and P. Nath, Phys. Rev. D **32** (1985) 2348.
4. S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. D **24** (1981) 1681.
5. B. Grinstein, Nucl. Phys. B **206** (1982) 206; J. Ellis, J.S. Hagelin, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. B **115** (1982) 380; I. Antoniadis, J. Ellis, J.S. Hagelin and D.V. Nanopoulos, Phys. Lett. B **194** (1987) 231.
6. M. Green, J. Schwarz and E. Witten, Superstring Theory, 2 vols., Cambridge University Press, 1987.
7. B. Greene *el al.*, Phys.Lett.**B180** (1986) 69; Nucl.Phys.**B278** (1986) 667; **B292** (1987) 606; R. Arnowitt and P. Nath, Phys.Rev.**D39** (1989) 2006; **D42** (1990) 2498; Phys.Rev.Lett. **62** (1989) 222.
8. M. Dine, N. Seiberg and E. Witten, Nucl.Phys.**B289** (1987) 585.
9. A.E. Faraggi, D.V. Nanopoulos and K. Yuan, Nucl. Phys. B **335** (1990) 347.
10. A.E. Faraggi, Nucl. Phys. B **387** (1992) 239.
11. A.E. Faraggi, Phys. Lett. B **278** (1992) 131.

12. A.E. Faraggi, Phys. Lett. B **274** (1992) 47.

13. A.E. Faraggi, Phys. Lett. B **302** (1993) 202.

14. A. Font, L.E. Ibañez and F. Quevedo, Phys.Lett.**B228** (1989) 79.

15. A.E. Faraggi, Nucl. Phys. B **403** (1993) 101.

16. I. Antoniadis, C. Bachas, and C. Kounnas, Nucl.Phys.**B289** (1987) 87; I. Antoniadis and C. Bachas, Nucl.Phys.**B298** (1988) 586; H. Kawai, D.C. Lewellen, and S.H.-H. Tye, Nucl.Phys.**B288** (1987) 1; R. Bluhm, L. Dolan, and P. Goddard, Nucl.Phys.**B309** (1988) 330.

17. I. Antoniadis, J. Ellis, J. Hagelin, and D.V. Nanopoulos, Phys. Lett. B **231** (1989) 65.

18. S. Kalara, J. Lopez and D.V. Nanopoulos, Phys. Lett. B **245** (1991) 421; Nucl. Phys. B **353** (1991) 650.

19. S. Dimopoulos, S. Raby and F. Wilczek, Phys. Lett. B **112** (1982) 133.

20. J. Ellis, J. Lopez and D.V. Nanopoulos, Phys. Lett. B **252** (1990) 53, G. Leontaris and T. Tamvakis, Phys. Lett. B **260** (1991) 333.

21. S. Kalara, J. Lopez and D.V. Nanopoulos, Phys. Lett. B **245** (1991) 421; Nucl. Phys. B **353** (1991) 650.

F	SEC	$SU(3)_C \times SU(2)_L$	Q_C	Q_L	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$SU(5) \times SU(3)$	Q_7	Q_8
D_1	$b_2 + b_3 + \beta$	(3,1)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	0	(1,1)	$-\frac{1}{4}$	$-\frac{15}{4}$
$\bar{D}_1$	$\pm\gamma + (I)$	($\bar{3}$,1)	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	0	(1,1)	$\frac{1}{4}$	$\frac{15}{4}$
D_2	$b_1 + b_3 + \alpha$	(3,1)	$\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	0	0	(1,1)	$-\frac{1}{4}$	$-\frac{15}{4}$
$\bar{D}_2$	$\pm\gamma + (I)$	($\bar{3}$,1)	$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	0	(1,1)	$\frac{1}{4}$	$\frac{15}{4}$
D_3	$1 + \alpha$	(3,1)	$\frac{1}{2}$	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	(1,1)	-1	0
$\bar{D}_3$	$+2\gamma$	($\bar{3}$,1)	$-\frac{1}{2}$	0	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	(1,1)	1	0
H_1	$b_2 + b_3 + \beta$	(1,1)	$-\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	0	0	(1,3)	$\frac{3}{4}$	$\frac{5}{4}$
$\bar{H}_1$	$\pm\gamma + (I)$	(1,1)	$\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	0	(1,$\bar{3}$)	$-\frac{3}{4}$	$-\frac{5}{4}$
H_2	$b_1 + b_3 + \alpha$	(1,1)	$-\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	0	0	(1,3)	$\frac{3}{4}$	$\frac{5}{4}$
$\bar{H}_2$	$\pm\gamma + (I)$	(1,1)	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	0	(1,$\bar{3}$)	$-\frac{3}{4}$	$-\frac{5}{4}$

Table 1. Massless states and their quantum numbers in the model of table 3. The first three pairs are additional color triplets. The last two pairs are the states with vanishing weak hypercharge and fractional $U(1)_{Z'}$ charge.

	ψ^μ	$\{\chi^{12};\chi^{34};\chi^{56}\}$	$\bar\psi^1, \bar\psi^2, \bar\psi^3, \bar\psi^4, \bar\psi^5, \bar\eta^1, \bar\eta^2, \bar\eta^3$	$\bar\phi^1, \bar\phi^2, \bar\phi^3, \bar\phi^4, \bar\phi^5, \bar\phi^6, \bar\phi^7, \bar\phi^8$
α	0	$\{0,\ 0,\ 0\}$	1, 1, 1, 0, 0, 0 , 0, 0	1, 1, 1, 1, 0, 0, 0, 0
β	0	$\{0,\ 0,\ 0\}$	1, 1, 1, 0, 0, 0, 0, 0	1, 1, 1, 1, 0, 0, 0, 0
γ	0	$\{0,\ 0,\ 0\}$	$\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$	$\frac{1}{2}$, 0, 1, 1, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, 0

	$y^3y^6, y^4\bar y^4, y^5\bar y^5, \bar y^3\bar y^6$	$y^1\omega^6, y^2\bar y^2, \omega^5\bar\omega^5, \bar y^1\bar\omega^6$	$\omega^1\omega^3, \omega^2\bar\omega^2, \omega^4\bar\omega^4, \bar\omega^1\bar\omega^3$
α	1, 0, 0, 0	0, 0, 1, 1	0, 0, 1, 1
β	0, 0, 1, 1	1, 0, 0, 0	0, 1, 0, 1
γ	0, 1, 0, 1	0, 1, 0, 1	1, 0, 0, 0

Table 2. A three generations $SU(3)\times SU(2)\times U(1)^2$ model, with $|\alpha_L(b_j)-\alpha_R(b_j)| = 1$, $(j=1,2,3)$. The color triplets from the Neveu–Schwarz sector are projected out and the electroweak triplets remain in the physical spectrum. The sector $b_1+b_2+\alpha+\beta$ produces one pair of electroweak doublets and one pair of color triplets. The 16 right–moving internal fermionic states $\{\bar\psi^{1,\cdots,5},\bar\eta^1,\bar\eta^2,\bar\eta^3,\bar\phi^{1,\cdots,8}\}$, correspond to the 16 dimensional compactified torus of the ten dimensional heterotic string. The 12 left–moving and 12 right–moving real internal fermionic states correspond to the six left and six right compactified dimensions in the bosonic language. ψ^μ are the two space–time external fermions in the light–cone gauge and χ^{12}, χ^{34}, χ^{56} correspond to the spin connection in the bosonic constructions.

	ψ^μ	$\{\chi^{12};\chi^{34};\chi^{56}\}$	$\bar\psi^1, \bar\psi^2, \bar\psi^3, \bar\psi^4, \bar\psi^5, \bar\eta^1, \bar\eta^2, \bar\eta^3$	$\bar\phi^1, \bar\phi^2, \bar\phi^3, \bar\phi^4, \bar\phi^5, \bar\phi^6, \bar\phi^7, \bar\phi^8$
α	0	$\{0,\ 0,\ 0\}$	1, 1, 1, 0, 0, 0, 0, 0	1, 1, 1, 1, 0, 0, 0, 0
β	0	$\{0,\ 0,\ 0\}$	1, 1, 1, 0, 0, 0, 0, 0	1, 1, 1, 1, 0, 0, 0, 0
γ	0	$\{0,\ 0,\ 0\}$	$\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$	$\frac{1}{2}$, 0, 1, 1, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, 0

	y^3y^6, $y^4\bar y^4$, $y^5\bar y^5$, $\bar y^3\bar y^6$	$y^1\omega^6$, $y^2\bar y^2$, $\omega^5\bar\omega^5$, $\bar y^1\bar\omega^6$	$\omega^1\omega^3$, $\omega^2\bar\omega^2$, $\omega^4\bar\omega^4$, $\bar\omega^1\bar\omega^3$
α	1, 1, 1, 0	1, 1, 1, 0	1, 1, 1, 0
β	0, 1, 0, 1	0, 1, 0, 1	1, 0, 0, 0
γ	0, 0, 1, 1	1, 0, 0, 0	0, 1, 0, 1

Table 3. A three generations $SU(3)\times SU(2)\times U(1)^2$ model. The color triplets from the Neveu–Schwarz sector are projected out and the electroweak triplets remain in the physical spectrum. The color triplets from the sector $b_1+b_2+\alpha+\beta$ are projected out as well. The notation used is the notation of table 2.

	ψ^μ	$\{\chi^{12};\chi^{34};\chi^{56}\}$	$\bar\psi^1, \bar\psi^2, \bar\psi^3, \bar\psi^4, \bar\psi^5, \bar\eta^1, \bar\eta^2, \bar\eta^3$	$\bar\phi^1, \bar\phi^2, \bar\phi^3, \bar\phi^4, \bar\phi^5, \bar\phi^6, \bar\phi^7, \bar\phi^8$
α	1	{1, 0, 0}	1, 1, 1, 0, 0, 1 , 0, 0	1, 1, 0, 0, 0, 0, 0, 0
β	1	{0, 1, 0}	1, 1, 1, 0, 0, 0, 1, 0	1, 1, 0, 0, 0, 0, 0, 0
γ	1	{0, 0, 1}	$\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$	$\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, 1, 0, 0, 0

	$y^3\bar y^3, y^4\bar y^4, y^5\bar y^5, y^6\bar y^6$	$y^1\bar y^1, y^2\bar y^2, \omega^5\bar\omega^5, \omega^6\bar\omega^6$	$\omega^1\bar\omega^1, \omega^2\bar\omega^2, \omega^3\bar\omega^3, \omega^4\bar\omega^4$
α	1, 0, 0, 1	0, 0, 1, 0	0, 0, 0, 1
β	0, 0, 0, 1	0, 1, 1, 0	1, 0, 0, 0
γ	1, 1, 0, 0	1, 0, 0, 0	0, 1, 0, 0

Table 4. A three generations $SU(3)\times SU(2)\times U(1)^2$ model, with $|\alpha_L(b_j)-\alpha_R(b_j)| = 0$, $(j=1,2,3)$. The electroweak doublets from the Neveu–Schwarz sector are projected out and the color triplets remain in the physical spectrum.

	ψ^μ	$\{\chi^{12};\chi^{34};\chi^{56}\}$	$\bar\psi^1, \bar\psi^2, \bar\psi^3, \bar\psi^4, \bar\psi^5, \bar\eta^1, \bar\eta^2, \bar\eta^3$	$\bar\phi^1, \bar\phi^2, \bar\phi^3, \bar\phi^4, \bar\phi^5, \bar\phi^6, \bar\phi^7, \bar\phi^8$
α	1	$\{1,\ 0,\ 0\}$	1, 1, 1, 0, 0, 1, 0, 1	1, 1, 1, 1, 0, 0, 0, 0
β	1	$\{0,\ 1,\ 0\}$	1, 1, 1, 0, 0, 0, 1, 1	1, 1, 1, 1, 0, 0, 0, 0
γ	1	$\{0,\ 0,\ 1\}$	$\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$	$\frac{1}{2}$, 0, 1, 1, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, 0

	$y^3\bar y^3, y^4\bar y^4, y^5\bar y^5, y^6\bar y^6$	$y^1\bar y^1, y^2\bar y^2, \omega^5\bar\omega^5, \omega^6\bar\omega^6$	$\omega^2\omega^3, \omega^1\bar\omega^1, \omega^4\bar\omega^4, \bar\omega^2\bar\omega^3$
α	1, 0, 0, 1	0, 0, 1, 0	0, 0, 1, 1
β	0, 0, 0, 1	0, 1, 1, 0	0, 1, 0, 1
γ	1, 1, 0, 0	1, 1, 0, 0	0, 0, 0, 1

Table 5. A three generations $SU(3)\times SU(2)\times U(1)^2$ model. The condition $|\alpha_L(b_j)-\alpha_R(b_j)| = 1$, is obeyed for $j=3$ but not for $j=1,2$. Therefore, we obtain the color triplets D_1, $\bar D_1$, D_2, $\bar D_2$, and the electroweak doublets h_3 and $\bar h_3$. The notation used is the notation of table 2.